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STATIC DEFORMATION DUE TO A LONG TENSILE FAULT OF FINITE WIDTH IN AN ISOTROPIC HALF-SPACE IN SMOOTH CONTACT WITH AN ORTHOTROPIC HALF-SPACE

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Abstract—*Closed-form analytical expressions for stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in smooth contact with a homogeneous, orthotropic, perfectly elastic half-space caused by a tensile fault of finite width located at an arbitrary distance from the interface in the isotropic half-space are obtained. The Airy stress function approach is used to obtain the expressions for the stresses.*

1. INTRODUCTION

The theory of elasticity gives an approximation to the stress-strain behaviour of real materials. Stresses and strains within the earth are important precursors of earthquakes. The elastic dislocation theory is an important tool to study sources of earthquakes, to model surface deformations for any slip distributions, in interpreting geodetic observations etc. It has become an important part of seismology since Steketee (1958) applied it to Geophysics. Since then, dislocation theories have been developed for different earth models. Many scientists derived analytical expressions to calculate co-seismic deformations for different earth models and fault types. Earth is being treated as an elastic body. Quite often, natural deposits in the earth are horizontally layered. The layered structure of it exerts a significant influence on the deformation field generated by a dislocation source. The elastic properties of the material may be different in different directions at a point of a layer i.e. the medium may be anisotropic. A two-phase medium is comprised of two half-spaces with different properties separated by a single plane boundary. A medium with three mutually perpendicular planes of elastic symmetry at a point is known as orthorhombic. Furthermore, if one of the plane of symmetry in an orthorhombic symmetry is horizontal, the medium is termed as orthotropic. This symmetry is exhibited by the principal rock-forming minerals of the deep crust and upper mantle e.g. olivine and orthopyroxenes. In orthotropic materials, there are nine elastic constants instead of two in isotropic materials.

Till now, many researchers have obtained the deformation field due to shear or tensile faults in the two half-space model e.g. Singh and Rani (1991), Singh et al. (1992), Bonafede and Rivalta (1999), Kumari et al. (2002), Kumar et al. (2005), Rani and Bala (2006), Bala and Rani (2009), Malik et al. (2012, 2014), Godara et al. (2014,2014) and others. The studies related to static deformation of half-spaces in smooth contact are scarce than that of in welded contact. Malik et al. (2014) studied the deformation of two isotropic, homogeneous, perfectly elastic half-spaces in smooth contact caused by a vertical tensile fault. Godara et al. (2014) replaced the lower isotropic half-space by the orthotropic half-space and obtained the expressions for stresses and displacements at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in smooth contact with a homogeneous, orthotropic, perfectly elastic half-space caused by a vertical dip-slip fault. In the present note, we consider the same model except the fault type. Our aim is to study the stress field caused by an inclined tensile fault of finite width located at an arbitrary distance from the interface in the isotropic half-space. The expressions for the stresses are obtained using the Airy stress function approach.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Consider a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space ($x_3 > 0$, Upper half-space) in smooth contact with an orthotropic half-space ($x_3 < 0$, Lower half − space) along the plane $x_3 = 0$. Let the lower edge of the tensile fault of finite width be located at the point (y_2, y_3) of the upper half-space at a distance d from the interface. The x_1-axis is taken parallel to the strike of the fault and x_3-axis vertically upwards. The stress-strain relations for the isotropic half-space are:

$$p_{ij} = 2\mu\left[e_{ij} + \frac{\sigma}{12\sigma}\delta_{ij}e_{kk}\right], \quad (i,j = 1,2,3) \quad (1)$$

where p_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, μ is the shear modulus and σ is Poisson's ratio.

The stress-strain relations for the orthotropic half-space are:

$$\begin{bmatrix} p'_{11} \\ p'_{22} \\ p'_{33} \\ p'_{23} \\ p'_{31} \\ p'_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e'_{11} \\ e'_{22} \\ e'_{33} \\ 2e'_{23} \\ 2e'_{31} \\ 2e'_{12} \end{bmatrix} \quad (2)$$

where p'_{ij}, e'_{ij} are stress and strain tensors of orthotropic medium respectively and c_{ij} are the coefficients of stiffness matrix for $i,j = 1,2,3$.

For a two dimensional approximation in which displacement components u_1, u_2, u_3 are independent of x_1 so that $\partial/\partial x_1 \equiv 0$, the plane strain problem $(u_1 = 0)$ and anti-strain problem $(u_2 = 0$ and $u_3 = 0)$ are decoupled and therefore, can be solved separately.

The plane strain problem for an isotropic medium can be solved in terms of Airy stress function U such that

$$p_{22} = \partial^2 U/\partial x_3^2, \quad p_{33} = \partial^2 U/\partial x_2^2, \quad p_{23} = -\partial^2 U/\partial x_2 \partial x_3 \quad (3)$$

$$\nabla^2\nabla^2 U = 0 \quad (4)$$

The plane strain problem for an orthotropic medium can be solved in terms of the Airy stress function U^* (Garg et al. (1991)) such that

$$p'_{22} = \partial^2 U^*/\partial x_3^2, \quad p'_{33} = \partial^2 U^*/\partial x_2^2,$$

$$p'_{23} = -\partial^2 U^*/\partial x_2 \partial x_3, \quad (5)$$

$$\left(a^2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right)\left(b^2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right)U^* = 0, \quad (6)$$

$$a^2 + b^2 = \frac{(c_{22}c_{33} - c_{23}^2 - 2c_{23}c_{44})}{c_{33}c_{44}}, \quad a^2b^2 = \frac{c_{22}}{c_{33}}. \quad (7)$$

For an isotropic medium,

$$c_{11} = c_{22} = c_{33} = \frac{2\mu(1-\sigma)}{1-2\sigma},$$

$$c_{12} = c_{13} = c_{23} = \frac{2\mu\sigma}{1-2\sigma},$$

$$c_{44} = c_{55} = c_{66} = \mu. \quad (8)$$

This yields $a^2 = b^2 = 1$ and equation (6) reduces to equation (4).

Godara et al. (2014) obtained the Airy stress function for an arbitrary line source parallel to the x_1-axis and passing through the point (y_2, y_3) located in the isotropic half-space in smooth contact with the orthotropic half-space.

For the isotropic half-space, we have

$$\begin{aligned} U = {} & L_0 \tan^{-1}\left(\frac{x_2 - y_2}{|x_3 - h|}\right) + M_0 \frac{(x_2 - y_2)|x_3 - y_3|}{R^2} \\ & - P_0 \ln R + Q_0 \frac{(x_3 - y_3)^2}{R^2} \\ & + L^-\left\{A_1 \tan^{-1}\left(\frac{x_2 - y_2}{x_3 + y_3}\right) - \frac{2A_2(x_2 - y_2)x_3}{S^2}\right\} \\ & + M^-\left\{\frac{(A_1 y_3 + x_3)(x_2 - y_2)}{S^2} - \frac{4A_2 x_3 y_3 (x_2 - y_2)(x_3 + y_3)}{S^4}\right\} \\ & + P^-\left\{-A_1 \ln S - \frac{2A_2 x_3(x_3 + y_3)}{S^2}\right\} \\ & + Q^-\left\{\frac{(A_1 y_3 + x_3)(x_3 + y_3) + 2A_2 x_3 y_3}{S^2} - \frac{4A_2 y_3 x_3 (x_3 + y_3)^2}{S^4}\right\} \end{aligned} \quad (9)$$

and for orthotropic half-space,

$$\begin{aligned} U^* = {} & 2L^-\left\{-B_2 \tan^{-1}\left(\frac{x_2 - y_2}{y_3 - ax_3}\right) + B_1 \tan^{-1}\left(\frac{x_2 - y_2}{y_3 - bx_3}\right)\right\} \\ & + 2M^-\left\{(x_2 - y_2)y_3\left(\frac{-B_2}{T^2} + \frac{B_1}{H^2}\right)\right\} \\ & - 2P^-\{-B_2 \ln T + B_1 \ln H\} \\ & + 2Q^-\left\{\frac{-B_2 y_3 (y_3 - ax_3)}{T^2} + \frac{B_1 y_3 (y_3 - bx_3)}{H^2}\right\} \end{aligned} \quad (10)$$

where,

$(x_2, x_3) =$ Receiver's location,

$R^2 = (x_2 - y_2)^2 + (x_3 - y_3)^2$,

$S^2 = (x_2 - y_2)^2 + (x_3 + y_3)^2$,

$T^2 = (x_2 - y_2)^2 + (y_3 - ax_3)^2$,

$H^2 = (x_2 - y_2)^2 + (y_3 - bx_3)^2$,

$x_3 \neq y_3\,,\quad x_2 \neq y_2\,,\quad ax_3 \neq y_3\,,\quad bx_3 \neq y_3,$

$$A_1 = \frac{(a-b) - 2\mu\alpha(as_2 - bs_1)}{G},$$

$$A_2 = \frac{2\mu\alpha(as_2 - bs_1)}{G},$$

$$B_1 = \frac{a}{G},\quad B_2 = \frac{b}{G},$$

$$G = (a-b) + 2\mu\alpha(as_2 - bs_1),$$

$$r_1 = \frac{c_{33}a^2 + c_{23}}{\Delta},\quad r_2 = \frac{c_{33}b^2 + c_{23}}{\Delta},$$

$$s_1 = \frac{c_{23}a^2 + c_{22}}{a\Delta},\quad s_2 = \frac{c_{23}b^2 + c_{22}}{b\Delta},$$

$$\Delta = (c_{22}c_{33} - c_{23}^2). \qquad (11)$$

In equations (9) and (10), L_0, M_0, P_0, Q_0 are the source coefficients and L^-, M^-, P^-, Q^- are the values of these source coefficients valid for $x_3 < y_3$. Singh and Garg (1986) and Singh and Rani (1991) have given these source coefficients for various seismic sources.

Using the values of the source coefficients L_0, M_0, P_0, Q_0, L^-, M^-, P^-, Q^- in equations (9) and (10), the Airy stress function due to a long tensile line source of arbitrary dip parallel to x_1-axis and acting at the point (y_2, y_3) located in the isotropic half-space in smooth contact with the orthotropic half-space in the form;

for isotropic half-space;

$$U = \frac{\alpha\mu b' ds}{\pi}\Big[-\ln R - A_1 \ln S - \frac{2A_2x_3(x_3+y_3)}{S^2} + \cos 2\delta\Big\{\frac{(x_3-y_3)^2}{R^2} + \frac{(A_1y_3+x_3)(x_3+y_3)+2A_2x_3y_3}{S^2} - \frac{4A_2x_3y_3(x_3+y_3)^2}{S^4}\Big\} - \sin 2\delta\Big\{\frac{(x_2-y_2)(x_3-y_3)}{R^2} - \frac{(A_1y_3+x_3)(x_2-y_2)}{S^2} + \frac{4A_2x_3y_3(x_2-y_2)(x_3+y_3)}{S^4}\Big\}\Big] \qquad (12)$$

and for the orthotropic half space;

$$U^* = \frac{2\alpha\mu b' ds}{\pi}\Big[B_2 \ln - B_1 \ln H + \cos 2\delta\Big\{-\frac{B_2y_3(y_3-ax_3)}{T^2} + \frac{B_1y_3(y_3-bx_3)}{H^2}\Big\} - \sin 2\delta\Big\{(x_2-y_2)y_3\Big(\frac{B_2}{T^2} - \frac{B_1}{H^2}\Big)\Big\}\Big] \quad (13)$$

where, $\alpha = \dfrac{1}{2(1-\sigma)}$.

Now, using the polar coordinates (s,δ) (see Figure 1),

$y_2 = s\cos\delta,$

$y_3 = d + s\sin\delta$

and integrating over s between the limits (0, L), we will obtain the following expressions for the Airy stress function for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface:

$$U = \frac{\alpha\mu b'}{\pi}\Big[\{(1+A_1)\cos^2\delta\}s + (x_2\cos\delta + X\sin\delta - s)\ln R + \{A_1(x_2\cos\delta - d\sin\delta - s) - x_3\sin\delta)\}\ln S + 2A_2x_3(x_2\cos\delta - X'\sin\delta - s)(d + s\sin\delta)\frac{1}{S^2} - x_3\cos\delta\,(A_1 + 2A_2 - 1)\,\tan^{-1}\Big(\frac{s - x_2\cos\delta + X'\sin\delta}{x_2\sin\delta + X'\cos\delta}\Big)\Big]\Big|_0^L \quad (14)$$

$$U^* = \frac{2\alpha\mu b'}{\pi}\Big[(B_1 - B_2)s\cos^2\delta - B_2(x_2\cos\delta - d\sin\delta - s)\ln T + B_1(x_2\cos\delta - d\sin\delta - s)\ln H - B_2ax_3\cos\delta\tan^{-1}\Big(\frac{s - x_2\cos\delta + Y\sin\delta}{x_2\sin\delta + Y\cos\delta}\Big) + B_1bx_3\cos\delta\tan^{-1}\Big(\frac{s - x_2\cos\delta + Y'\sin\delta}{x_2\sin\delta + Y'\cos\delta}\Big)\Big]\Big|_0^L \qquad (15)$$

where now,

$R^2 = (x_2 - s\cos\delta)^2 + (X - s\sin\delta)^2,$

$S^2 = (x_2 - s\cos\delta)^2 + (X' + s\sin\delta)^2,$

$T^2 = (x_2 - s\cos\delta)^2 + (Y + s\sin\delta)^2,$

$H^2 = (x_2 - s\cos\delta)^2 + (Y' + s\sin\delta)^2,$

$X = x_3 - d,\quad X' = x_3 + d,$

$Y = d - ax_3,\quad Y' = d - bx_3\,,$

$f(s)|_0^L = f(L) - f(0).$

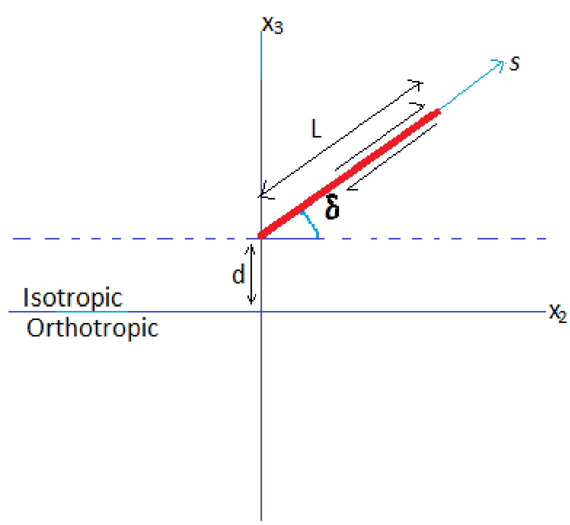

Figure 1. Geometry of a tensile fault of width L having lower edge at a distance *d* from the interface in the isotropic half-space in smooth contact with an orthotropic half-space where *δ* is dip angle and *s* is the distance from the lower edge of the fault, measured in dip direction.

3. STRESS FIELD

Using equations (3) and (14), we obtain the following expressions of stresses for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface as for isotropic half-space,

$$p_{22} = \frac{\alpha\mu b'}{\pi}\Big[(x_2\cos\delta + 3X\sin\delta + s\cos 2\delta) - 2(x_2\cos\delta + X\sin\delta - s)(X - s\sin\delta)^2\frac{1}{R^4} + \{2(x_2\cos\delta - X'\sin\delta - s) - (4A_2 + A_1)(x_2\cos\delta + d\sin\delta - s\cos 2\delta) - x_2\sin\delta\}\frac{1}{S^2} - 2\{A_1(x_2\cos\delta - X'\sin\delta - s)(X' + s\sin\delta)^2 - 2A_2x_3^2\cos\delta\,(x_2 - s\cos\delta) - (A_1 - 1)x_3(X' + s\sin\delta)(x_2\cos\delta - X'\sin\delta - s) + 2A_2(d + s\sin\delta)(X' + s\sin\delta)[2(x_2\cos\delta - X'\sin\delta - s) + 3x_3\sin\delta]\}\frac{1}{S^4} + 16A_2x_3(x_2\cos\delta - X'\sin\delta - s)(d + s\sin\delta)(X' + s\sin\delta)^2\frac{1}{S^6}\Big]\Big|_0^L \tag{16}$$

$$p_{23} = \frac{\alpha\mu b'}{\pi}\Big[-(x_2\sin\delta - X\cos\delta)\frac{1}{R^2} + 2(X - s\sin\delta)^2(x_2\sin\delta - X\cos\delta)\frac{1}{R^4} + \{-(A_1 + 4A_2 - 1)x_3\cos\delta + (x_2\sin\delta + X'\cos\delta)\}\frac{1}{S^2} + \{2x_3(A_1 + 2A_2)(X' + s\sin\delta)^2\cos\delta) - 2A_1(X' + s\sin\delta)[X'\cos\delta(X' + s\sin\delta) + d\sin\delta(x_2 - s\cos\delta) + s\cos\delta(x_2\sin\delta + X'\cos\delta)] - 4A_2(d + s\sin\delta)[(x_3 + X' + s\sin\delta)(x_2\sin\delta + X'^{\cos\delta}) + 2x_3\cos\delta(X' + s\sin\delta)] - 2x_3(X' + s\sin\delta)(x_2\sin\delta + X'\cos\delta)\}\frac{1}{S^4} + 16A_2x_3(d + s\sin\delta)(X' + s\sin\delta)^2(x_2\sin\delta + X'\cos\delta)\frac{1}{S^6}\Big]\Big|_0^L \tag{17}$$

$$p_{33} = \frac{\alpha\mu b'}{\pi}\Big[(x_2\cos\delta - X\sin\delta - s\cos 2\delta)\frac{1}{R^2} + 2(x_2\cos\delta + X\sin\delta - s)(X - s\sin\delta)^2\frac{1}{R^4} + \{A_1(x_2\cos\delta + d\sin\delta - s\cos 2\delta) + x_3\sin\delta\}\frac{1}{S^2} + \{2A_1(X' + s\sin\delta)[(X' + s\sin\delta)(x_2\cos\delta - d\sin\delta - s) - x_3\cos\delta\,(x_2 - s\cos\delta)] - 4A_2x_3[x_3\cos\delta\,(x_2 - s\cos\delta) + 3\sin\delta\,(X' + s\sin\delta)(d + s\sin\delta)] + 2x_3(X' + s\sin\delta)(x_2\cos\delta - X'\sin\delta - s)\}\frac{1}{S^4} - 16A_2x_3(x_2\cos\delta - X'\sin\delta - s)(d + s\sin\delta)(X' + s\sin\delta)^2\frac{1}{S^6}\Big]\Big|_0^L \tag{18}$$

Using equations (5) and (18), we obtain the following expressions of stresses for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface as for orthotropic half-space,

$$p'_{22} = \frac{2\alpha\mu b'}{\pi}\Big[-a^2B_2(s\cos 2\delta - x_2\cos\delta - d\sin\delta)\frac{1}{T^2} + b^2B_1(s\cos 2\delta - x_2\cos\delta - d\sin\delta)\frac{1}{H^2} + 2a^2B_2(d + s\sin\delta)(x_2\cos\delta - Y\sin\delta - s)(Y + s\sin\delta)\frac{1}{T^4} - 2b^2B_1(d + s\sin\delta)(x_2\cos\delta - Y'^{\sin\delta - s})(Y' + s\sin\delta)\frac{1}{H^4}\Big]\Big|_0^L$$

(19)

$$p'_{23} = \frac{2\alpha\mu b'}{\pi}\Big[a^2B_2x_3\cos\delta\frac{1}{T^2} - b^2B_2x_3\cos\delta\frac{1}{H^2} + 2aB_2(d + s\sin\delta)(-x_2\sin\delta - Y\cos\delta)(Y + s\sin\delta)\frac{1}{T^4} - 2bB_1(d + s\sin\delta)(-x_2\sin\delta - Y'cos\delta)(Y' + s\sin\delta)\frac{1}{H^4}\Big]\Big|_0^L \tag{20}$$

$$p'_{33} = \frac{2\alpha\mu b'}{\pi}\Big[-B_2(x_2\cos\delta + d\sin\delta - s\cos 2\delta)\frac{1}{T^2} + B_1(x_2\cos\delta + d\sin\delta - s\cos 2\delta)\frac{1}{H^2} - 2B_2(d + s\sin\delta)(x_2\cos\delta - Y\sin\delta - s)(Y + s\sin\delta)\frac{1}{T^4} + 2B_1(d + s\sin\delta)(x_2\cos\delta - Y'sin\delta - s)(Y' + s\sin\delta)\frac{1}{H^4}\Big]\Big|_0^L \tag{21}$$

4. DISCUSSION

Equations (14) - (21) yield the Airy stress function and stress field at any point of a two-phase medium consisting of an isotropic half-space overlying the orthotropic half-space due to an inclined tensile fault placed at distance d from the interface in the isotropic half-space. These closed-form analytical expressions are very convenient for computing the stresses at any point of the medium. It has been examined that when the lower half-space is replaced by the isotropic one, then the results of the present paper, in the limit, coincide with the corresponding results for the stresses of Malik et al. (2014) for two half-spaces to be in smooth contact. It has also been verified that the stresses obtained satisfy the necessary continuity conditions at the interface.

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